NOTATION

I_λ, $J_{e,\lambda}$, spectral radiative intensity of the medium and of a perfect blackbody; $\mu = \cos\theta$; θ, angle between the direction of the y axis, normal to the surface, and the ambient direction of propagation of the radiation; k_λ, volume spectral absorption coefficient; $(\ )_y$, differentiation with respect to y; T, gas temperature; σ, Stefan–Boltzmann constant; λ, wavelength of the radiation; R, radius of blunting of the body; p, pressure in the shock layer; H, stagnation enthalpy; q_R, integral radiative heat flux to the surface; q_c, convective heat flux; h, shock layer thickness.

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SPATIAL NONSTATIONARY HEAT-CONDUCTION PROBLEM FOR A PRISM WITH A COORDINATE-DEPENDENT HEAT-TRANSFER COEFFICIENT

Yu. M. Kolyano and E. G. Grits'ko

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We present an efficient method for the determination of three-dimensional non-steady-state fields of bodies of simple shapes, when the heat-transfer coefficient from their surface changes locally.

We consider an isotropic semiinfinite rectangular prism $0 \leqslant z \leqslant \infty$, $0 \leqslant x \leqslant x_0$, $0 \leqslant y \leq y_0$. Through the face z = 0 of the prism, convective heat exchange takes place with the inhomogeneous external medium. The temperature of the external medium, in contact with an arbitrary region Γ of the z = 0 surface is equal to t_{m1}. The remaining part of the surface z = 0 is in contact with an external medium of temperature t_m. The heat-transfer coefficient in the region Γ is denoted by α_1, and from the surface z = 0 outside Γ by α, with $\alpha_1 > \alpha$. The surfaces $x = 0$, $x = x_0$, $y = 0$, $y = y_0$ are either thermally insulated or are kept at temperature t_m. In dimensionless variables, the boundary-value problem for the determination of the non-steady-state temperature field in the semiinfinite rectangular prism can be written as

$$\frac{\partial^2\theta}{\partial Z^2} + \frac{\partial^2\theta}{\partial X^2} + \frac{\partial^2\theta}{\partial Y^2} = \frac{\partial\theta}{\partial\,\mathrm{Fo}}, \tag{1}$$

$$\beta_x \left.\frac{\partial\theta}{\partial X}\right|_{x=0} = \gamma_x\theta|_{x=0},\ \ \beta_x \left.\frac{\partial\theta}{\partial X}\right|_{x=\pi} = -\gamma_x\theta|_{x=\pi}, \tag{2}$$

$$\beta_y \left.\frac{\partial\theta}{\partial Y}\right|_{y=0} = \gamma_y\theta|_{y=0},\ \ \beta_y \left.\frac{\partial\theta}{\partial Y}\right|_{y-b} = -\gamma_y\theta|_{y=b},$$

$$|\beta_\zeta|\,|\gamma_\zeta| = 0,\ \ |\beta_\zeta| + |\gamma_\zeta| \neq 0,\ \ \zeta = X,\ Y,$$

$$\frac{\partial\theta}{\partial Z} = \mathrm{Bi}\,\theta + [(\mathrm{Bi}_1 - \mathrm{Bi})\,\theta - \mathrm{Bi}]\,\chi(X,\ Y)\ \ \text{for}\ \ Z = 0, \tag{3}$$

$$\theta|_{t\to\infty} = 0,\ \ \theta|_{\mathrm{Fo}\to 0} = 0, \tag{4}$$

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where

$$\theta=(t-t_{\rm m})/(t_{\rm m1}-t_{\rm m});\ X=\pi x/x_0;\ \ Y=\pi y/x_0;\ \ Z=\pi z/x_0;\ \ b=\pi y_0/x_0;\ \ \mathrm{Fo}=\pi^2\tau a/x_0^2;\ \mathrm{Bi}_1=\alpha_1 x_0(\lambda\pi)^{-1};\ \mathrm{Bi}=\alpha x_0(\lambda\pi)^{-1};$$

$$\chi(X,\ Y)=\begin{cases}1 & \text{for } X,\ Y\in\Gamma;\\ 0 & \text{for } X,\ Y\bar{\in}\Gamma.\end{cases}$$

We take the Laplace transform of (1) and (3) with respect to Fo and finite, cosine or sine, Fourier transforms [1] with respect to the X and Y coordinates. Using (2) and (4), we then obtain

$$\frac{d^2\hat{\theta}}{dZ^2}=\nu\hat{\theta}, \tag{5}$$

$$\frac{d\hat{\theta}}{dZ}-\mathrm{Bi}\hat{\theta}=-\frac{\mathrm{Bi}_1}{s}\bar{\chi}(x,\ y)+(\mathrm{Bi}_1-\mathrm{Bi})\iint\limits_{\Gamma}\tilde{\theta}\Phi_e(nX)\,\Phi_\mu(\xi Y)\,dXdY$$
$$\text{for } Z=0,\ \ \hat{\theta}\big|_{z\to\infty}=0. \tag{6}$$

Here,

$$\hat{\theta}=\int\limits_0^\pi\int\limits_0^b\tilde{\theta}\Phi_e(nX)\,\Phi_\mu(\zeta Y)\,dXdY;\ \xi=k\pi/b;\ \ \nu=\sqrt{n^2+\xi^2+s};\ \ \tilde{\theta}=\int\limits_0^\infty\theta\times\exp(-s\mathrm{Fo})\,d\mathrm{Fo};\ \Phi_0(\zeta)=\cos\zeta;\ \Phi_1(\zeta)=\sin\zeta;\ \ \bar{\chi}(x,\ y)=\iint\limits_{\Gamma}\Phi_e(nX)\times\Phi_\mu(\xi Y)\,dXdY.$$

Analogously to [2, 3] we replace the Laplace transform of the temperature field in the integrand in (6) by its integral characteristic $\tilde{\vartheta}$ in the region Γ, i.e.,

$$\tilde{\theta}\big|_{y,X\in\Gamma}\approx\tilde{\vartheta}, \tag{7}$$

where

$$\tilde{\vartheta}=\frac{1}{P}\iint\limits_{\Gamma}\tilde{\theta}^*dXdY. \tag{8}$$

Here θ^* is the temperature field in the semiinfinite rectangular prism obtained by substitution (7), and P is the area of the region Γ in units x_0^2/π^2.

The solution of the boundary-value problem (5) and (6), taking into account (7), is

$$\hat{\theta}^*=[\mathrm{Bi}_1/s-(\mathrm{Bi}_1-\mathrm{Bi})\tilde{\vartheta}]\,\varphi(Z)\,\bar{\chi}(X,\ Y), \tag{9}$$

where $\varphi(Z)=\exp(-\nu Z)(\nu+\mathrm{Bi})^{-1}$.

Taking the inverse sine or cosine Fourier transform of (9) and substituting the result into (8), we find

$$\tilde{\vartheta}=\frac{1}{s}\ \frac{S_\vartheta\mathrm{Bi}_1P^{-1}}{1+S_\vartheta(\mathrm{Bi}_1-\mathrm{Bi})P^{-1}}. \tag{10}$$

Here,

$$S_\vartheta=\sum_{n=l}^{\infty}\ \sum_{k=\mu}^{\infty}\varepsilon(n,\ k)\,[\bar{\chi}(x,\ y)]^2\,\varphi(0),$$

$$\varepsilon(n,\ k)=\begin{cases}4(\pi b)^{-1} & \text{for } nk\neq 0,\\ 2(\pi b)^{-1} & \text{for } nk=0 \text{ and } n+k\neq 0,\\ (\pi b)^{-1} & \text{for } k+n=0.\end{cases}$$

Substituting (10) into (9) and taking inverse Fourier and Laplace transforms of the resulting (9), we obtain

$$\theta^* = \frac{\mathrm{Bi}_1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty} f(s)\,\frac{\exp(s\,\mathrm{Fo})}{s}\,ds, \tag{11}$$

where

$$f(s) = \sum_{n=l}^{\infty}\sum_{k=\mu}^{\infty}\varepsilon(n,\ k)\,\varphi(Z)\,\chi(X,\ Y)\,\Phi_e(nX)\,\Phi_\mu(\xi Y)\ [1 + S_\vartheta(\mathrm{Bi}_1 - \mathrm{Bi})]^{-1}. \tag{12}$$

To reduce the difficult contour integral (11) to a Riemann integral, we use the following result. If the complex numbers v_n satisfy for all n = 1, 2, 3, ..., the condition

$$|\arg v_n| \leqslant \frac{\pi}{2}, \tag{13}$$

then the following relation holds:

$$\left|\arg\left(\sum_{n=0}^{\infty} v_n\right)\right| \leqslant \frac{\pi}{2} \tag{14}$$

provided the sum exists.

If we consider expression (12) for s in the region $|\arg s| \leqslant \pi$ and noting the above result one can conclude that f(s) has no poles in this region. The contour of integration in the contour integral (11) can therefore be taken along the two sides of a cut along the real negative semiaxis, and round the point s = 0.

Transforming the obtained integral, we obtain

$$\theta^* = \mathrm{Bi}_1\left[f(0) + \frac{1}{\pi}\int_0^{\infty}\frac{\exp(-\eta\,\mathrm{Fo})}{\eta}\,\frac{(L_{1d} + L_1^{\infty})L_{0u} - (1 + L_{0d} + L_0^{\infty})L_{1d}}{(1 + L_{0d} + L_0^{\infty})^2 + L_{0u}^2}\,d\eta\right], \tag{15}$$

where

$$L_{1d} = \sum_{n=l}^{E(\sqrt{\eta})}\ \sum_{k=\mu}^{E(\sqrt{\eta - n^2})} M(n,\ k,\ X,\ Y)(\mathrm{Bi}\cos Z\nu_u + \nu_u \sin Z\nu_u)(\mathrm{Bi}^2 + \nu_u^2)^{-1};$$

$$L_{1u} = \sum_{n=l}^{E(\sqrt{\eta})}\ \sum_{k=\mu}^{E(\sqrt{\eta - n^2})} M(n,\ k,\ X,\ Y)(\mathrm{Bi}\sin Z\nu_u + \nu_u \cos Z\nu_u)(\mathrm{Bi}^2 + \nu_u^2)^{-1};$$

$$L_1^{\infty} = \sum_{n=E_x}^{\infty}\ \sum_{k=E_y}^{\infty} M(n,\ k,\ X,\ Y)\exp(-\nu_d Z)(\nu_d + \mathrm{Bi})^{-1};$$

$$L_{0d} = C\,\mathrm{Bi}\sum_{n=l}^{E(\sqrt{\eta})}\ \sum_{k=\mu}^{E(\sqrt{\eta - n^2})} M_1(n,\ k)(\mathrm{Bi}^2 + \nu_u)^{-1};$$

$$L_{0u} = C\sum_{n=l}^{E(\sqrt{\eta})}\ \sum_{k=\mu}^{E(\sqrt{\eta - n^2})} M_1(n,\ k)\,\nu_u(\mathrm{Bi}^2 + \nu_u^2)^{-1};$$

$$L_0^{\infty} = C\sum_{n=E_x}^{\infty}\ \sum_{k=E_y}^{\infty} M_1(n,\ k)(\nu_d + \mathrm{Bi})^{-1};$$

$$\nu_d = \sqrt{n_x^2 + \xi^2 - \eta}\ ;\ \nu_u = \sqrt{\eta - n^2 - \xi^2};\ E_x = E(\sqrt{\eta}) + 1;$$

$$E_y = E(\sqrt{\eta - n^2}) + 1;\ M(n,\ k,\ X,\ Y) = \varepsilon(n,\ k)\,\bar{\chi}(X,\ Y)\times$$

$$\times\Phi_l(nX)\,\Phi_\mu(\xi Y);\ M_1(n,\ k) = \varepsilon(n,\ k)\,[\bar{\chi}(X,\ Y)]^2;$$

and $C = (\mathrm{Bi}_1 - \mathrm{Bi})P^{-1}$, $E(\zeta)$ is the integer part of ξ.

The quantities L_{1d}, L_{1u}, L_{0d}, L_{0u} should be set equal to zero in the following cases:

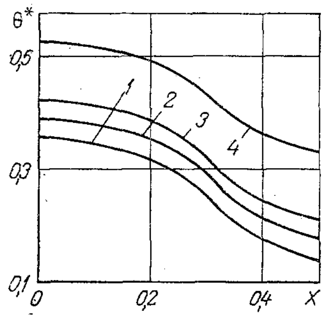

Fig. 1. Dimensionless temperature field θ^* as a function of the dimensionless coordinate X for all values of the dimensionless coordinate Y at Z = 0.

a) $l = 1$, $\mu = 0$, $\eta \leqslant 1$;

b) $l = 0$, $\mu = 1$, $\eta \leqslant \pi/b^2$;

c) $l = 1$, $\mu = 1$, $\eta \leqslant 1 + \pi/b^2$.

The solution (15) is meaningful for μ, $l = 0, 1$, and is applicable to the following situations:

a) thermally insulated side surface of the semiinfinite prism ($\mu + l = 0$);

b) the side surface of the prism is kept at temperature t_c ($\mu l = 1$);

c) two opposite edges of the side surface of the semiinfinite prism are kept at temperature t_c, and the other two are insulated ($\mu + l = 1$).

The accuracy of solution (15) can be estimated by using the error functions introduced in [4].

Calculations carried out according to formulas (15) with the region of the local thermal interaction chosen in the form of a rectangle $\varepsilon_{ox} \leqslant X \leqslant \varepsilon_{1x}$, $\varepsilon_{oy} \leqslant Y \leqslant \varepsilon_{1y}$, showed that the temperature field in the semiinfinite prism when the side surface is thermally insulated, reaches its steady-state value considerably slower than when its side surface was kept at zero temperature. The values of the steady-state temperature field in the semiinfinite prism with thermally insulated side surface, assuming other conditions being identical, considerably exceed the values of the temperature field when its side surface is kept at zero temperature. Figure 1 shows the functional dependence of the temperature field on the X coordinate in an arbitrary cross section Y = const for $z = 0$, $\varepsilon_{oy} = 0$, $\varepsilon_{1y} = b$, $\varepsilon_{ox} = 0$, $\varepsilon_{1x} = 0.3$, and Fo = 3, 10, 30, ∞ (curves 1-4), and $\mu = l = 0$.

NOTATION

t, temperature field; θ, dimensionless temperature field; x, y, and z, dimensional coordinates; X, Y, and Z, dimensionless cordinates; τ, time; λ, thermal conductivity; and a, thermal diffusivity.

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